A Convolutionally Coded Orthogonal Multicarrier DS/CDMA System in Time Limited Asynchronous Channels

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Abstract—In this paper, we propose to transmit convolutionally coded DS waveforms over orthogonally overlapped subchannels in time limited asynchronous channels. The bit error rate of the decoder output is investigated along with simulation results under several conditions on the CSI. It is shown that the proposed system, the convolutionally coded orthogonal multicarrier DS/CDMA system, significantly outperforms the system using frequency diversity combining. It is also shown that the proposed system has better performance than the convolutionally coded almost non-overlapped multicarrier DS/CDMA system under the condition that the information rate and total available bandwidth are the same for both systems.

I. INTRODUCTION

Recently, multicarrier transmission schemes have been introduced into CDMA systems to get such advantages as the bandwidth efficiency, frequency diversity, and interference rejection capability [1]-[4] in high data rate transmission. Among them, some systems in which narrowband DS waveforms are transmitted over a number of overlapped or non-overlapped subchannels were proposed as alternatives to the rake receiver to mitigate the effects of frequency selective channels [2]-[4]. A non-overlapped multicarrier DS/CDMA system with maximal ratio combining was proposed in [2]. With such a scheme, we can reduce the interference from other subchannels. Convolutional code was applied to this system to improve the performance [1]. On the other hand, an orthogonal multicarrier DS/CDMA system with equal gain combining was proposed in [3]. With orthogonally overlapped subchannels, the bandwidth can be used efficiently.

In this paper, a convolutionally coded orthogonal multicarrier (CC/OM) DS/CDMA system is proposed in time limited asynchronous channels. In the proposed system, convolutionally coded bits are transmitted over a number of subchannels instead of identical bits being sent as in [3]. We analyze the statistical characteristics of the correlator outputs of the proposed system and compare them with those of the convolutionally coded almost non-overlapped multicarrier (CC/ANM) DS/CDMA system of which the subcarrier spacing is a multiple of the subchannel bandwidth (since we consider the time limited system, the tails of subchannel spectra slightly overlap). Since the interference variance of each subchannel can be different, we consider three decoding metrics for the decoder according to the channel state information (CSI) of fading amplitudes and noise variances. We will get a tight bound on the bit error rate (BER) in Rayleigh fading channel when all the CSI is ideally available. Under other conditions, we will resort to simulations results. The performance of the proposed system under each condition on the CSI is compared with that of the diversity combining orthogonal multicarrier (DC/OM) DS/CDMA system. It is also compared with the performance of the CC/ANM DS/CDMA system under the condition that the information rate and total available bandwidth are fixed.

II. SYSTEM MODEL

Fig. 1 shows the power spectral density (psd) of time limited multicarrier DS signals for the two kinds of multicarrier DS/CDMA systems. The psd of ANM DS signals is shown in Fig. 1(a), where the frequency spacing between successive subcarriers is \( \frac{2}{T_c} \), with \( T_c \) the chip duration of the ANM DS/CDMA system. If we define the total transmission bandwidth \( BW_T \) as the null-to-null bandwidth, we have \( BW_T = \frac{2M}{T_c} \), where \( M \) is the number of subcarriers of the ANM DS/CDMA system. The psd of OM DS signals is shown in Fig. 1(b), in which the subcarrier spacing is \( \frac{1}{T_c} \) and total bandwidth is \( BW_T = \frac{M+1}{T_c} \), with \( T_c \) the chip duration and \( M \) the number of subcarriers of the OM DS/CDMA system.

The system model for the CC/OM DS/CDMA system proposed in this paper is shown in Fig. 2. The information bit streams \( b_k \), duration \( T_b \), are encoded by a rate \( 1/M \) convolutional code encoder. The \( M \) coded binary symbols \( \{ x_{k,m} \} \) are allocated to \( M \) subchannels simultaneously at time \( t \).
frequency diversity. They are interleaved and then spread by a pseudo noise (PN) code waveform \( c(t) = \sum_{n=0}^{N-1} c_{k,n} p(t - nT_c) \), where \( c_{k,n} = \pm 1 \) is the spreading chip, \( T_c = T_b/N \) is the chip duration, \( N \) is the processing gain of DS narrowband waveforms modulated by orthogonal subcarriers, and \( p(t) = 1 \) for \( 0 < t < T_b \) and \( p(t) = 0 \), otherwise.

Then the transmitted signal \( s_k(t) \) of user \( k \) can be written as

\[
s_k(t) = \sqrt{2P} \sum_{i=-\infty}^{\infty} \sum_{m=1}^{M} a_{k,m} x_i^{k,m} c_k(t - iT_b) \cos(\omega_m t + \varphi_{k,m}),
\]

where \( P \) is the transmitted power per subcarrier, \( \{\varphi_{k,m}\}_{m=1}^{M} \) are random phases of subcarriers, \( \omega_m = 2\pi f_m \) is the carrier angular frequency of the \( m \)-th subchannel.

The channel is assumed to be frequency selective Rayleigh fading and not to vary during one bit duration. However, the subchannels are assumed to be nonselective by choosing the number of subcarriers appropriately as in [2]. Then the complex lowpass impulse response of the subchannels of user \( k \) can be modeled as

\[
h_{k,m}(t) = a_{k,m} e^{j\beta_{k,m} \delta(t)}, \text{ for } m = 1, 2, \ldots, M,
\]

where \( a_{k,m} \) is the fading amplitude and \( \beta_{k,m} \) is the random phase of the subchannel. The fading amplitudes \( \{a_{k,m}\}_{m=1}^{M} \) are in general correlated, but we will assume that they are independent and identically distributed (i.i.d.) Rayleigh random variables by interleaving the code symbols. The phases \( \{\beta_{k,m}\}_{m=1}^{M} \) are also i.i.d. random variables uniform over \([0, 2\pi)\).

Let us assume that there are \( K \) users in a cell and power control is employed. Then the received signal at the base station can be written as

\[
r(t) = \sqrt{2P} \sum_{i=-\infty}^{\infty} \sum_{k=1}^{K} \sum_{m=1}^{M} a_{k,m} x_i^{k,m} c_k(t - \tau_k - iT_b) \cos(\omega_m t + \varphi_{k,m}) + n(t),
\]

where the propagation delays \( \{\tau_k\}_{k=1}^{K} \) are i.i.d. random variables uniform over \([0, T_b)\), \( \varphi_{k,m} = (\varphi_{k,m} + \beta_{k,m} - \omega_m \tau_k) \mod 2\pi \) are also i.i.d. random variables uniform over \([0, 2\pi]\), and \( n(t) \) is the additive white Gaussian noise (AWGN) with mean zero and variance \( N_0/2 \).

The received signal is coherently demodulated by the subcarrier and then correlated by each user's code waveform. Let the first user be the desired user, \( \tau_1 = 0 \), and current time bit be \( b_{k,m}^0 \). Then the correlator output of the \( q \)-th subcarrier of the desired user is given by

\[
Y_q = \int_{0}^{T_b} r(t) c_1(t) \cos(\omega_q t + \varphi_{1,q}) dt.
\]

After the correlator output \( Y_q \) is deinterleaved, the output goes through a soft decision Viterbi decoder properly weighted according to the CSI.

We can similarly describe the CC/ANM DS/CDMA system except for the subcarrier spacing. To compare the performance of the CC/ANM and CC/OM DS/CDMA systems, we fix the bandwidth expansion ratio of both systems such that

\[
F = \frac{\text{Total bandwidth of a system}}{\text{Total bandwidth of a system without coding and spreading}} = \frac{M+N'}{M+N},
\]

where \( N' \) is the processing gain of the CC/ANM DS/CDMA system.

III. PERFORMANCE ANALYSIS

A. Analysis of correlator outputs

The correlator output of the \( q \)-th subchannel can be expressed as

\[
Y_q = S_q + U_q + A_q + N_q,
\]

where \( S_q = \sqrt{P/2} T_b x_{i,q}^{1,q} \) is the desired signal, \( U_q \) is the interference from other users in the same subchannel, \( A_q \) is the interference from adjacent subchannels, and \( N_q \) is the correlator output of the AWGN.

The co-channel interference \( U_q \) and the adjacent subchannel interference \( A_q \) can be written as

\[
U_q = \sqrt{P/2} \sum_{k=2}^{K} \sum_{q=1}^{M} a_{k,q} \cos(\Delta \varphi_{k,q}^{1,q})
\cdot \left[ x_{k,q}^{1,q} R_{k,1}(\tau_k) + x_{k,q}^0 R_{k,1}(\tau_k) \right]
\]

where \( \Delta \varphi_{k,q}^{1,q} = \varphi_{k,q} - \varphi_{1,q} \).
In this paper, we use random sequences as the spreading sequences so that $I_q$ can be assumed to be a Gaussian random variable from the central limit theorem. Then (13) can be written as

$$\sigma_q^2 = \frac{K - 1}{3N} + \frac{K - 1}{2\pi^2N} \sum_{m=1}^{M} \frac{1}{(m-q)^2} + \frac{N_q}{2E_b}. \quad (14)$$

Similarly, we can obtain the variance of $I_q$ for the CC/ANM DS/CDMA system. Note that the variance of $I_q$ can be different for different values of $q$ when there exist other users.

B. Decoding Metrics

Now we investigate the performance of the soft decision Viterbi decoder. Let $y = \langle y_1, \ldots, y_M \rangle$ be the truncated received codeword, where $y_i$ is the $i$-th correlator output at time index $i$ after deinterleaving. When all the CSI (the fading amplitudes and noise variances of all subchannels) is ideally available, the maximum likelihood decoding metric for the path codeword $x = \langle x_1, \ldots, x_M \rangle$ can be expressed as

$$m(y, x; \alpha, \sigma^2) = -\sum_{i=1}^{L} \sum_{q=1}^{M} \frac{|y_i - \alpha_q x_i|^2}{\sigma_q^2}. \quad (15)$$

When the information on the noise variance is not available, we can consider the decoding metric

$$m(y, x; \alpha) = -\sum_{i=1}^{L} \sum_{q=1}^{M} |y_i - \alpha_q x_i|^2. \quad (16)$$

When neither fading amplitude nor noise variance is available, the decoding metric is

$$m(y, x) = -\sum_{i=1}^{L} \sum_{q=1}^{M} |y_i - x_i|^2. \quad (17)$$

Once a metric is chosen based on the availability of the CSI, the soft decision Viterbi decoder chooses the coded sequence for which the metric is maximized.

C. Bound for bit error probability

In this subsection, we analyze the system performance when the decoding metric (15) is used. Since it is difficult to find the exact BER of the decoder output, we will instead obtain a tight bound for the BER using the method proposed in [6] for Rayleigh fading channels.

The conditional pairwise probability for the decoder to select a competing path $\hat{x}$ when the correct path codeword is $x$ can be written as

$$P_2(x, \hat{x} \mid \alpha) = \Pr \left\{ \sum_{i=1}^{L} \sum_{q=1}^{M} \frac{\alpha_q^2 (x_i - \hat{x}_i)^2}{\sigma_q^2} > 0 \right\} = Q \left( \sum_{(i,q) \in \eta} \frac{(\alpha_q^2)^2}{\sigma_q^2} \right), \quad (18)$$
where \( \eta \) is the set of \((l, q)\) for which \( x^1_l \neq x_q^1 \) and \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx \). Then the the pairwise probability is obtained by averaging (18) over \( \alpha \) as

\[
P_2(x, \hat{x}) = \int_0^\infty \int_0^\infty Q(\sqrt{2}d(x, \hat{x}|\alpha))P_\alpha(\alpha_1)\ldots P_\alpha(\alpha_M)d\alpha, \tag{19}
\]

where \( d(x, \hat{x}|\alpha) = \sqrt{\sum_{(l, q) \in \eta} (a^l_q)^2 / 2\sigma^2_q} \) and \( P_\alpha(\alpha) = 2\alpha e^{-\alpha^2} \), for \( \alpha > 0 \).

Let us define a new vector

\[
a = (a_1^1, \ldots, a_M^1, \ldots, a_1^d, \ldots, a_M^d), \tag{20}
\]

where \( a_q^l = a_q^l \sqrt{1 + \gamma_q} \) with \( \gamma_q = 1 / 2\sigma^2_q \) the average signal to noise ratio (SNR). Then

\[
P_2(x, \hat{x}) = \prod_{(l, q) \in \eta} \int_0^\infty \int_0^\infty Q(\sqrt{2}d(x, \hat{x}|\alpha))e^{i\alpha}P_\alpha(\alpha_1)\ldots P_\alpha(\alpha_M)d\alpha, \tag{21}
\]

where \( d(x, \hat{x}|\alpha) = \sqrt{\sum_{(l, q) \in \eta} (a^l_q)^2 w_q^2} \) and \( w_q = \sqrt{\gamma_q / (1 + \gamma_q)} \).

Since \( \gamma_{\min} = \gamma_1 \leq \gamma_q \leq \gamma_M = \gamma_{\max} \), it follows that

\[
w_{\min} z \leq d(x, \hat{x}|\alpha)^2 \leq w_{\max}^2, \tag{22}
\]

where \([\cdot]\) represents the largest integer not greater than \( \cdot \), \( w_{\min} = \sqrt{\gamma_{\min} / (1 + \gamma_{\min})} \), \( w_{\max} = \sqrt{\gamma_{\max} / (1 + \gamma_{\max})} \), and \( z = \sum_{(l, q) \in \eta} (a^l_q)^2 \).

Therefore the pairwise error probability of the two paths can be bounded as

\[
P_2(x, \hat{x}) \leq G(d_{\text{free}}, w_{\min}) \prod_{(l, q) \in \eta} \frac{1}{\sqrt{1 + \gamma_q}}, \tag{23}
\]

where \( d_{\text{free}} \) is the free distance of the given convolutional code and

\[
G(d, w) = \int_0^\infty Q(w \sqrt{z})e^{w^2 z} P_2(z)dz
= \frac{1}{2\pi(d-1)!} \sum_{j=1}^d B_j^{(d)} \frac{1}{(1+w)^j}. \tag{24}
\]

In (24), \( P_2(z) \) is the pdf of \( z \) and \( B_j^{(d)} \) can be evaluated by the following recursive relations:

For \( d = 1 \), \( B_1^{(1)} = 1 \)

For \( d \geq 1 \),

\[
B_1^{(d)} = B_2^{(d)} = (2d-3)!! = (2d-3)(2d-5) \cdots 1
B_j^{(d)} = 2 \left[ B_{j-1}^{(d)} - (d-1)B_{j-2}^{(d)} \right], \quad j = 3, 4, \ldots, d. \tag{25}
\]

To find the upper bound on the bit error probability using (23), we should find the transfer function of the convolutional code: the transfer function should not only generate the number of nonzero bits but also show where the code symbol is assigned to among the subchannels. If \( D_q \) is the nonzero code bit that is assigned to the \( q \)-th subchannel and \( I \) is the information symbol, the upper bound on the bit error probability can be found by

\[
P_b \leq G(d_{\text{free}}, w_{\min}) \left| \frac{\partial T(D_1, D_2, \ldots, D_M, I)}{\partial I} \right|_{I=1, D_q = \frac{1}{\sqrt{1+\gamma_q}}} \tag{26}
\]

where \( T(\cdot) \) is the transfer function of the given convolutional code as found in [1].

## IV. Performance Evaluation

In this section, we investigate the performance of the proposed CC/OM DS/CDMA system. In the investigation, we fix the bandwidth expansion ratio as

\[
F = N \left( \frac{M+1}{2} \right) = M' M'' = 480. \tag{27}
\]

Monte Carlo method is used for the simulation with 1 million runs for each point when the BER is higher than \( 10^{-4} \) and 50 million runs for each point when the BER is lower than \( 10^{-4} \). For convolutional coding, we use rate \( 1/M \) maximum free distance convolutional code with constraint length 3 [7].

Fig. 3 shows the bound (26), Chernoff bound on the BER, and simulation results when all the CSI is available and ideal, \( F = 480, M = 4, \) and \( K = 1 \) and \( 100 \).

The BER curves versus \( E_b/N_0 \) of the proposed CC/OM DS/CDMA system are shown under the three conditions on the CSI together with those of the DC/OM DS/CDMA system in Fig. 4, when \( K = 100, M = 4, \) and \( N = 192 \). In the figure, the decoder in the CC/OM DS/CDMA system uses the decoding metric (15) for all CSI, metric (16) for partial CSI, and metric (17) for no CSI.
The combiner in the DC/OM DS/CDMA system uses the max-
imal ratio combining weight \( w_q = \alpha_{1,q}/\sigma_q^2 \) for all CSI, partial combining weight \( w_q = \alpha_{1,q} \) for partial CSI, and equal gain combining weight \( w_q = 1 \) for no CSI to obtain the decision variable \( T = \sum_{q=1}^{M} w_q y_q \), where \( y_q = \alpha_{1,q}b_q^T + i_q \) is the correlator output. It is observed that the performance of the CC/OM DS/CDMA system is considerably improved when compared with that of the DS/OM DS/CDMA system. Since there exists only slight difference in the performance between the cases all CSI and partial CSI, the decoding metric (16) can be a good substitute for (15).

Fig. 5 shows the BER versus the number of users when \( E_b/N_o = 10dB \) under the same conditions as in Fig. 4. It is observed that about 150 users can be admitted at BER = 10\(^{-3}\) and about 30 users at BER = 10\(^{-5}\) in the CC/OM DS/CDMA system when all the CSI is available and ideal.

The analytical bound on the BER and simulation results of the CC/OM and CC/ANM DS/CDMA systems are shown as functions of the number of users in Fig. 6 when \( E_b/N_o = 10dB \), the number of subcarriers is 4 and 7. It is observed that the performance gain of the CC/OM DS/CDMA system over the CC/ANM DS/CDMA system increases as the numbers of users and subcarriers increase.

V. CONCLUDING REMARK

In this paper, we proposed a CC/OM DS/CDMA system where convolutionally coded symbols were transmitted over a number of orthogonally overlapped subchannels. We analyzed the statistical characteristics of the correlator outputs and obtained a tight bound on the BER of the decoding output for Rayleigh fading channels when all the CSI was available and ideal. Under the other conditions on the CSI, we resorted to simulation for the performance evaluation. It was shown that the CC/OM DS/CDMA system significantly outperformed the DC/OM DS/CDMA system. When the information rate and total available bandwidth were fixed, it was observed that the proposed CC/OM DS/CDMA system outperformed the CC/ANM DS/CDMA system.

ACKNOWLEDGEMENTS

This research was supported by Korea Science and Engineering Foundation under Grant 981-0915-078-2, for which the authors would like to express their thanks.

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