Decoding of MIMO Systems with Hypothesis Testing Technique
M.A. Jeong, D. Kim, J. Oh, H. Yuxi, H.-K. Min, and I. Song (Korea)

T-DMB GIS-AEAS Receiver Model

A Low-Complexity Sphere Decoder with Noise-Predictive Radius Initialization
J.L. Laguna Morales and S. Roy (Canada)

Chaotic MIMO — Spatial Diversity Techniques for DCSK Modulation
J.L. Laguna Morales and S. Roy (Canada)

Robust Joint Channel Estimation and Symbol Detection via Sphere Decoding
J.L. Laguna Morales and S. Roy (Canada)

On the Double Doppler Effect Generated by Scatterer Motion
V.-H. Pham, M. Haj Taieb, J.-Y. Chouinard, S. Roy (Canada), and H.-T. Huỳnh (Vietnam)
DECODING OF MIMO SYSTEMS WITH HYPOTHESIS TESTING TECHNIQUE

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ABSTRACT
In this paper, we propose a near maximum likelihood (ML) decoding scheme for multiple input multiple output (MIMO) systems. Based on the multiple hypothesis testing problem, the proposed decoding scheme provides a higher efficiency than other conventional near ML decoding schemes by using some characteristics of the channel matrix. Numerical results show that, despite the proposed scheme has a lower computational complexity than other near ML decoders, the performance difference between the ML and proposed scheme is negligibly small.

KEY WORDS
MIMO, tree search, metric-first search, near ML

1 Introduction
The MIMO systems can provide high spectral efficiency and increased capacity, which are simply unattainable via conventional single input single output (SISO) techniques [1]-[3]. Due to the increased interference resulting from higher degree of spatial multiplexing when the number of antennas increases, on the other hand, the decoding task of MIMO systems is considerably more burdensome than that of SISO systems: in short, a more challenging decoding task is unavoidable to fully make use of the advantages of MIMO systems. The problem of designing efficient decoding schemes has been considered in many studies [3]-[5] for MIMO systems. Two representative ML decoders, the sphere decoder (SD) [4] and breadth-first signal decoder [5] have been proposed and shown to have the optimal bit error rate (BER) performance while allowing lower computational complexity. Despite a reduced computational complexity, the decoding task with the computational complexity of ML decoders is still quite more burdensome in practical systems than that of near ML decoders.

In order to achieve even lower computational complexity for decoders of MIMO systems, several near ML decoding schemes perform QR decomposition (QRD) of the channel matrix, use a tree structure to exploit a property of the upper triangular matrix, and regard the decoding problem as a problem of searching for a lattice point with the smallest node metric by employing the depth-, breadth-, and metric-first search methods on a tree.

Among a variety of near ML decoders employing the depth-first search method, Schnorr-Euchner2 (SE2) [7] and increasing radii algorithm (IRA) [9] schemes have been proposed to alleviate the exponentially growing computational complexity of ML schemes when the number of antennas increases. The QRD-M [6] scheme, a decoding scheme based on the breadth-first search method, reduces the search space by searching only $M$ nodes with the smallest node metrics in each layer. The efficient QRD-M [8] scheme, in which additional nodes are discarded when the node metric is larger than the square of the Euclidean distance of the partial decision feedback equalizer (DFE) solution in each layer, has been shown to have a lower computational complexity than the QRD-M scheme. As a metric-first search method, the QRD-Stack [10] reduces the computational complexity by using the stack algorithm [12]. Recently, the DELTA scheme reduces the computational complexity by employing the branch length threshold and SE enumeration, and has been shown to have a lower computational complexity than other near ML decoders. The decoding with expected length and threshold approximated (DELTA) [11] scheme reduces the computational complexity by employing the branch length threshold and SE enumeration, and has been shown to have a lower computational complexity than other near ML decoders.

In this paper, we propose a novel near ML decoder for MIMO systems which offers lower computational complexity than conventional decoders while maintaining the BER close to that of the ML decoder. The proposed decoder, a metric-first based scheme, first selects a node with the smallest node metric and then extends only one branch from the node by using pre-determined boundaries: the boundaries are obtained by applying the technique of multiple hypothesis testing problem [13]-[15]. In order to
maintain the BER close to the optimal performance, the proposed decoder then extends additional branches from the node. The number of additional branches is determined by comparing the absolute values of the diagonal elements of the upper triangular matrix \( \hat{R} \) of the channel matrix with pre-determined thresholds: the thresholds are obtained from a target signal to noise ratio (SNR) determined empirically.

2 System model

The block diagram of an MIMO system with \( N_T \) transmit and \( N_R \) receive antennas is illustrated in Fig. 1. It is assumed that a common quadrature amplitude modulation (QAM) is employed for all sub-stream transmitted from \( N_T \) transmit antennas. Then, the complex received signal vector \( \tilde{r} = [\tilde{r}_1, \tilde{r}_2, \cdots, \tilde{r}_{N_R}]^T \) can be expressed as

\[
\tilde{r} = \hat{H} \tilde{s} + \tilde{n},
\]

where \( \tilde{r}_j \) is the received signal at the \( j \)-th receive antenna, \( \tilde{s} = [\tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_{N_T}]^T \) is the transmitted signal vector with the entry \( \tilde{s}_i \) denoting the transmitted signal at the \( i \)-th transmit antenna, \( \tilde{n} = [\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_{N_R}]^T \) is a vector of independent and identically distributed (i.i.d.) complex additive Gaussian random variables with mean zero and variance \( \sigma^2 \), and the superscript \( T \) indicates the vector transpose. It is assumed in (1) that the wireless channel, represented by the \( N_R \times N_T \) complex channel transfer matrix \( \hat{H} \), is assumed to be a rich-scattering and flat Rayleigh fading channel, where the elements \( \{\hat{h}_{j,i}\} \) of the complex fading transfer matrix \( \hat{H} \) are i.i.d. complex Gaussian random variables with mean zero and variance one: this assumption is used for convenience in most of the studies (e.g., [4]-[11]). We assume that the estimation of the channel matrix \( \hat{H} \) has been completed before the decoding of signals at the receiver.

Transforming the matrix and vectors in the complex baseband model (1) into real expressions, we have the real received signal vector

\[
\begin{align*}
\mathbf{r} & = [r_1, r_2, \cdots, r_N]^T \\
& = \begin{pmatrix} 
\Re(\tilde{r}) \\
\Im(\tilde{r})
\end{pmatrix}
\end{align*}
\]

where \( H \) is the real counterpart of the complex channel transfer matrix \( \hat{H} \), and \( \Re(\cdot) \) and \( \Im(\cdot) \) indicate the real and imaginary parts, respectively. In (2), \( \tilde{s} = [s_1, s_2, \cdots, s_M]^T \) is the real transmitted signal vector, \( \tilde{n} = [n_1, n_2, \cdots, n_N]^T \) is the vector of real i.i.d. additive Gaussian noise with mean zero and variance \( \sigma^2/2 \), \( M = 2N_T \), and \( N = 2N_R \). For simplicity, it is assumed in this paper that \( N = M \) without loss of generality. Assuming that the entry \( \tilde{s}_i \) of \( \tilde{s} \) is drawn from an \( L^2 \)-QAM constellation, we have \( \tilde{s} \in S_L^M \), where

\[
S_L = \{e_1, e_2, \cdots, e_L\}
\]

is the real signal constellation of size \( L \) and \( S_L^M \) is the set of \( M \)-dimensional real vectors of the transmitted signal with \( e_k = -L\frac{k-1}{L-1} + k - 1 \). Here, we will assume the \( L^M \) possible elements \( \tilde{s}_1, \tilde{s}_2, \cdots, \tilde{s}_{L^M} \) of \( S_L^M \) are equally likely.

Performing the QRD on the channel transfer matrix \( H \), we can rewrite (2) as

\[
y = QR\tilde{s} + w,
\]

where \( y = Q^T \mathbf{r} = [y_1, y_2, \cdots, y_M]^T \) and \( w = Q^T \tilde{n} \). Note that the statistical properties of the noise term \( w \) in (5) are the same as those of \( \tilde{n} \) in (2) since \( QTQ = I \).

Exploiting the upper triangular property of the matrix \( R \), an \( L \)-ary tree with \( (M+1) \) layers starting from a root located in the \((M+1)\)-st layer, the highest layer, is used frequently [4]-[11] to find the ML or near ML solution in the decoding of MIMO systems. In such a scenario, a branch between the \((j+1)\)-st and \( j \)-th layers of the tree denotes a possible value of the \( j \)-th element \( s_j \) of the real transmitted signal vector \( \tilde{s} \), and a node in the tree denotes the vector of the branches in the unique path connecting the node and root. We will denote the \( k \)-th node in the \( j \)-th layer by the \((M-j+1)\)-dimensional vector \( \tilde{s}^{(k)}_j = [s^{(k)}_{j,j}, s^{(k)}_{j,j+1}, \cdots, s^{(k)}_{j,M}]^T \) for \( j = 1, 2, \cdots, M \) and \( k = 1, 2, \cdots, L^{M-j+1} \) with \( s^{(k)}_{j,j} \) denoting the branch between the \((l+1)\)-st and \( l \)-th layers in the unique path between \( \tilde{s}^{(k)}_j \) and the root: clearly, \( \tilde{s}^{(1)}_{M+1} \) denotes the root.

Let us define some terminology before we describe the proposed decoding scheme in detail. A leaf node is a node not connected to any of the nodes in the lower layer. A best node is a node with the smallest node metric among the leaf nodes. The neighborhood nodes (NN) of order \( v \) for the node \( \tilde{s}^{(k)}_j \) are the set

\[
A_j^{(k)}(v) = \left\{ \tilde{s}_j^{\max(k-v, L[\frac{j}{L}] - L+1)} \right\},
\]
of nodes around $y_k^{(v)}$ for $v = 0, 1, \ldots, L-1$, where $\lfloor x \rfloor$ denotes the smallest integer no smaller than $x$, $\max(x, y) = x$ when $x \geq y$ and $y$ when $x < y$, and $\min(x, y) = -\max(-x, y)$. It is noteworthy in (6) that $k-v \leq L \left\lfloor \frac{v}{L} \right\rfloor$ and $k+\nu \geq L \left\lfloor \frac{\nu}{L} \right\rfloor - L + 1$ since $L \left\lfloor \frac{\nu}{L} \right\rfloor = iL$ when $(i-1)L < k \leq iL$: therefore, $\min(k + v, L \left\lfloor \frac{\nu}{L} \right\rfloor) \geq \max(k - v, L \left\lfloor \frac{v}{L} \right\rfloor - L + 1)$ implying that $A_j^{(v)}(v)$ is always a non-empty set.

3 Proposed scheme

3.1 Preliminaries

Assume that the channel transfer matrix $H$, number $M$ of antennas, size $L$ of the signal constellation $S_L$, and real received signal vector $\mathbf{r}$ are given. Then, performing the QRD on $H$, we obtain $R, Q,$ and $\bar{w}$. 

3.1.1 Applying multiple hypothesis testing problem

Employing the technique of multiple hypothesis testing problem, let us first depict how we can choose one branch from the best node among $L$ possibilities.

Since the $j$-th transmitted signal $s_j$ takes on one of the $L$ values in $S_L$, the decoding problem for $s_j$ can be modelled as an $L$-ary hypothesis testing problem in which the hypotheses \{ $H_j^{(1)}$ $\}_{1 \leq 1}^L$ are described by

$$H_j^{(1)} : s_j = \frac{L-1}{2}$$

$$H_j^{(2)} : s_j = \frac{L-3}{2}$$

$$\vdots$$

$$H_j^{(L)} : s_j = \frac{L-1}{2}.$$  

Next, let us obtain the test statistic of the $L$-ary hypothesis testing problem based on the observation $\mathbf{y} = [y_1, y_2, \ldots, y_M]^T$ shown in (5). Rewriting the $j$-th observation

$$y_j = \sum_{l=j}^M r_{j,l}s_l + \sum_{l=j+1}^M r_{j,l}s_l + w_j$$

as the sum of the transmitted signal $s_j$ and noise, we have

$$y_j - \sum_{l=j+1}^M r_{j,l}s_l = s_j + \sum_{l=j+1}^M \frac{w_j}{r_{j,l}},$$

where the Gaussian noise $\frac{w_j}{r_{j,l}}$ has mean zero and variance $\sigma^2_{\bar{w}_{j,l}}$. When $j = M$ in (9), it is clear that we can use $\frac{w_M}{r_{j,M}}$ as the test statistic and decode the transmitted signal $s_M$.

Next, when $j = M - 1$, the left-hand side of (9) can be evaluated by replacing $s_M$ with the decoded value and thus can be used as the test statistic in the decoding of $s_{M-1}$. In essence, the left-hand side of (9) can be used as the test statistic in the decoding of $s_j$ when $\{ s_k \}_{k=j+1}^M$ are replaced with the decoded values:

$$\Lambda(z_{j+1}^{(p)}) = \frac{y_j - \sum_{l=j+1}^M r_{j,l}s_l^{(p)}}{r_{j,j}}$$

is a rational choice as the test statistic when we are on node $z_{j+1}^{(p)}$ in the tree for $p = 1, 2, \ldots, L-1$.

Under the assumption that $L$ possible elements of $S_L$ are equally likely and the noise zero-mean symmetric, if $\Lambda(z_{j+1}^{(p)})$ is between $e_{z_{j+1}^{(p)}} - e_{z_{j+1}^{(p)}}$ and $e_{z_{j+1}^{(p)}} + e_{z_{j+1}^{(p)}}$, where $e_0 = -\infty$ and $e_{L+1} = +\infty$, we would accept the $z$-th hypothesis: that is, we include the $z$-th branch in the searching path by extending the $z$-th branch from the node $z_{j+1}^{(p)}$ among the $L$ possibilities. In other words, the decision region $D_{z_{j+1}^{(p)}}$ of hypothesis $H_{z_{j+1}^{(p)}}$ is

$$D_{z_{j+1}^{(p)}} = \left\{ \Lambda(z_{j+1}^{(p)}) : T_{(z)} < \Lambda(z_{j+1}^{(p)}) \leq T_{(z+1)} \right\}$$

for $z = 1, 2, \ldots, L$, where

$$T_{(u)} = \frac{e_u + e_{u+1}}{2}$$

$$= \frac{(-L+1)(u-1) + (-L+1) + u}{2}$$

$$= \frac{2u - L}{2}$$

with $T_{(0)} = -\infty$ and $T_{(L)} = +\infty$ are the boundaries. Note that $T_{(u)}$ the average of the $u$-th and $(u+1)$-st elements of $S_L$.

3.1.2 Determining the number of additional branches

We now describe how to extend the additional branches from the best node while keeping the computation complexity as low as possible. Under the assumption of zero-mean symmetric noise, it is rather straightforward to see that, when we are to connect nodes in addition to $z_{j+1}^{(p)}$, the NN of $z_{j+1}^{(p)}$ are a reasonable choice. Clearly, the order of the NN will influence both on the computational complexity and BER performance. We determine the order of the NN in such a way that the resulting BER performance is close to the ML performance: in short, the order $v_j$ is determined to be $v_j$ if

$$\Upsilon^{(v_j)}(\bar{\sigma}) \leq |r_{j,j}| \leq \Upsilon^{(v_j)}(\bar{\sigma}),$$

where \{ $\Upsilon^{(u)}(\bar{\sigma})$ $\}_{u=0}^L$ as

$$\Upsilon^{(u)}(\bar{\sigma}) = \frac{2\beta(\bar{\sigma})}{2\beta - 1}$$

643
with \( \Upsilon^{(L)}(\bar{\sigma}) = \infty \), \( \Upsilon^{(L)}(0) = 0 \), and

\[
\beta(\bar{\sigma}) = \frac{\sigma}{\sqrt{2}} Q^{-1} \left( 1 - \frac{1}{2} \bar{P}_{BER, ML}(\bar{\sigma}) \right).
\]  

(15)

Here, the target standard deviation \( \bar{\sigma} \) is set to the value of \( \sigma \) at the required maximum BER and \( \bar{P}_{BER, ML}(\bar{\sigma}) \), that is, the optimal BER is obtained previously as the average over 10\(^6\) runs through computer simulations based on the ML decoder.

In summary, the proposed decoder first selects the best node and then extends one branch from the best node by finding the interval to which \( \Lambda(\mathbf{z}^{(p)}_{j+1}) \) belongs with the boundaries \( T(u) \) obtained from the technique of multiple hypothesis testing problem. Then, to maintain the BER close to the optimal BER, additional branches are extended from the best node: the number of additional branches extended is determined by using \( \{r_{j,j}\} \) and pre-determined value \( \{\Upsilon^{(L)}(\bar{\sigma})\}_{u=0}^{L} \).

### 3.2 The proposed decoding scheme

Given the channel transfer matrix \( H \), the number \( M \) of layers, the size \( L \) of the signal constellation \( S_L \), and the real received signal vector \( r_z \), we determine the boundaries \( T(u) \) from (12) and thresholds \( \Upsilon^{(L)}(\bar{\sigma}) \) from (14).

Then we perform QRD on \( H \) to obtain \( R, Q \), and \( y \), and then determine the orders \( \{v_j\}_{j=1}^{M} \) satisfying (13) by comparing the absolute values \( \{|r_{j,j}|_{j=1}^{M}\} \) of \( R \) with the thresholds \( \{\Upsilon^{(L)}(\bar{\sigma})\}_{u=0}^{L} \).

#### 3.2.1 Step 1: Initialization

Start searching the tree from the root: that is, let \( j = M \) and \( p = 1 \).

#### 3.2.2 Step 2: Computing the test statistic \( \Lambda(\mathbf{z}^{(p)}_{j+1}) \)

Compute the test statistic \( \Lambda(\mathbf{z}^{(p)}_{j+1}) \) for \( \mathbf{z}^{(p)}_{j+1} \) by (10).

#### 3.2.3 Step 3: Determining the integer \( \bar{z} \) with \( T(u) \)

Obtain the integer \( \bar{z} \) satisfying

\[
T_{(z-1)} < \Lambda(\mathbf{z}^{(p)}_{j+1}) \leq T_{(z)}
\]

(16)

with the boundaries \( T(u) \), and choose the \( z \)-th node \( \mathbf{z}^{(pL-(L-z))}_{j} \) among \( L \) possibilities in the \( j \)-th layer.

#### 3.2.4 Step 4: Making leaf nodes

Make new leaf nodes the NN of order \( v_j \) for the node \( \mathbf{z}^{(pL-(L-z))}_{j} \) obtained in Step 3 and then compute the node metric of the leaf nodes.

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**Figure 2.** An example of the tree searching with the proposed decoding scheme with metric-first search method.

### 3.2.5 Step 5: Determining the best node

Determine the best node \( \mathbf{z}^{(i)}_{b} \). If the best node \( \mathbf{z}^{(i)}_{b} \) is not in the first layer, in other words, if \( b \neq i, \ j = b - 1 \), and then we take Step 2. On the other hands, if the best node \( \mathbf{z}^{(i)}_{b} \) is in the first layer, output \( \mathbf{z}^{(i)}_{1} \). An example depicting the application of the proposed decoding scheme when \( M = 4 \) and \( L = 4 \) is shown in Fig. 2, where the number inside a node denotes the order of the formation of the node.

### 4 Numerical results

Let us now evaluate the computational complexity and BER performance of the proposed and other decoders. In the simulations, it is assumed that all symbols are transmitted with equal energy \( E_s = E_{tot}/N_s \) over a rich-scattering and flat Rayleigh fading channel, where \( E_{tot} \) is the total energy used over one symbol duration at the transmitter. Then the SNR at each receive antenna is expressed as \( \frac{E_s}{N_s} \). We evaluate and compare the BER performance and computational complexities of the proposed decoder, DELTA [11], QRD-Stack [10], efficient QRD-M [8], SE2 [7], and IRA [9] for several QAM constellations and numbers of antennas in the simulations. The performance of the ML decoder, SD [4], is also considered and compared with the near-ML decoders.

The BER performance as a function of the SNR for various numbers of antennas and sizes of signal constellation is shown in Fig. 3. In DELTA, QRD-Stack, and efficient QRD-M, the number of nodes retained is set equally to \( L^2 \) in the \( L^2 \)-QAM constellation so that DELTA, QRD-Stack, and efficient QRD-M can also exhibit near ML performance. In SD, the initial radius was obtained by the decision feedback equalizer (DFE) algorithm. In proposed decoder, the target standard deviation is set to the noise standard deviation at 25dB and 30dB in 16- and 64-QAM, respectively, when both \( N_T = N_R = 2 \) and 4. The solid,
dashed, and dotted lines are used to signify the proposed decoder, SD, and near ML decoders, respectively. We can observe that the BER performance of the proposed decoder and other near ML decoders are practically the same and close to the optimal BER performance.

Figs. 4–7 show the average number of multiplications as a function of the SNR for various numbers of antennas and sizes of signal constellation. It is observed that the proposed decoder has lower computational complexity than other near ML decoders in terms of the average number of multiplications. In addition, the gain in the computational complexity of proposed decoder is more noticeable at low SNR and is quite robust to the variations of the size of signal constellation and SNR.

5 Conclusion

In this paper, we have proposed a novel near maximum likelihood decoder that provides significant gain in the computational complexity compared to conventional decoders, while maintaining the bit error rate close to the optimal performance. The proposed decoder first selects the best node and then extends one branch from the best node by the boundaries \( \{ T(u) \}_{u=0} \) obtained by applying the
technique of multiple hypothesis testing problem. Then, to maintain the BER close to the optimal BER, additional branches are extended from the best node using the absolute values of the diagonal elements of the upper triangular matrix $R$ of the channel matrix with thresholds. Numerical results show that the proposed decoding scheme has lower computational complexity than other near maximum likelihood decoders, while the performance difference between the maximum likelihood and proposed decoders is negligibly small.

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