A Three-Dimensional Distributed Source Modeling and Direction of Arrival Estimation Using Two Linear Arrays

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SUMMARY A number of results on the estimation of direction of arrival have been obtained based on the assumption that the signal sources are point sources. Recently, it has been shown that signal source localization can be accomplished more adequately with distributed source models in some real surroundings. In this paper, we consider modeling of three-dimensional distributed signal sources, in which a source location is represented by the center angles and degrees of dispersion. We address estimation of the elevation and azimuth angles of distributed sources based on the proposed distributed source modeling in the three-dimensional space using two linear arrays. Some examples are included to more explicitly show the estimation procedures under the model: numerical results obtained by a MUSIC-based method with two uniform linear arrays are discussed.

**key words:** three-dimensional distributed signal source, distributed source density, parametric distributed source model

1. Introduction

Many results on the direction of arrival (DOA) estimation have been obtained for the azimuth-only estimation problems based on the assumption that the signal sources are point sources [1]–[3]: i.e., if the DOA of a signal is \( \phi \), there is no signal at \( \phi + \epsilon \), for an arbitrarily small value of \( |\epsilon| \). The point source assumption is a reasonable one if the signal sources are located far enough from the receivers. Under this assumption, the array output vector is a weighted sum of a finite number of steering vectors when the number of signal sources is finite, with the weighting dependent on the signal sources. As an extension of the azimuth-only estimation problem, two-dimensional (azimuth and elevation) estimation problems have also been considered in several studies under the point source model [4], [5].

In physical surroundings, on the other hand, the signals received at an array usually consist not only of a direct path signal, but also of multiple return signals that are coherent, phase-delayed, and amplitude-weighted replicas of the direct path signal. Among such examples are multiple echoes in sonar, spurious returns such as clutter in radar, and coherent interference due to jamming signals in satellite communication. In such cases, the signal source direction is spread around the center direction \( \phi \), with multiple return signals existing in the interval \( [\phi - \epsilon_d, \phi + \epsilon_d] \) on a single frequency for some nonnegligible positive value of \( \epsilon_d \). We would like to mention that the coherent multipath signals sometimes arrive from quite different angles from the direct signal and thus any model depends on the environments in which the signal sources exist: in essence, the modeling is not unique.

It is noteworthy that the array output cannot be expressed as a weighted sum of a finite number of steering vectors when the signal sources are distributed: i.e., if the point source assumption is not valid, the array output vector should be expressed by integrating a steering vector over all directions of arrival with the weighting of distributed source density function [6]–[8].

In this paper, we consider simultaneous estimation of the elevation and azimuth angles of distributed signal sources: the results in this paper thus differ from those in [e.g., 4, 5] in that we consider distributed sources, not point sources. In addition, this paper is different from [6]–[8] in that we consider the estimation of both azimuth and elevation angles. We consider a method to estimate the azimuth and elevation angles of distributed signal sources based on the multiple signal classification (MUSIC), and the performance of the method is investigated with some examples.

2. The Estimation Problem and Signal Source Model

2.1 Estimation of DOA

Consider the azimuth-only estimation problem under the point source model, in which we have \( M \) array elements and \( L \) signal sources. The \( M \times 1 \) array snapshot (or array output) vector \( y(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T \) can be expressed as
also been considered in [e.g., 4, 5] by extending (2) as point signal sources in the three-dimensional space has sources are point sources, some aspects of the azimuth-φ = [φ1, φ2, ..., φL]T, the steering vector of the array depending on the array structure, and φk is the azimuth DOA of the kth source. The additive noise n(t) is assumed to be a stationary zero-mean white complex Gaussian random vector with E[n(t)nH(t)] = σ2I and E[n(t)nT(t)] = 0. The noise is assumed to be uncorrelated with the signal waveforms.

Denoting \( A(\phi) = [a(\phi_1), a(\phi_2), ..., a(\phi_L)] \) and \( \phi = [\phi_1, \phi_2, ..., \phi_L]^T \), (1) can be rewritten as

\[
y(t) = A(\phi)s(t) + n(t),
\]

where the zero-mean complex normal signal vector \( s(t) = [s_1(t), s_2(t), ..., s_L(t)]^T \) is stationary with covariance matrix \( E[s(t)s^H(t)] = R_s \) and \( E[s(t)s^T(t)] = 0 \). The steering vectors at different DOAs are linearly independent: i.e., the matrix \( A(\phi) \) has full rank \( L \).

Under these assumptions, the array output vector is complex Gaussian with mean zero and covariance matrix

\[
R_y = E[yy^H(t)] = A(\bar{\phi})R_sA^H(\phi) + \sigma^2I,
\]

where \( H \) denotes the Hermitian transpose. When the sources are point sources, some aspects of the azimuth-only estimation problems have been studied in [1]–[3].

Estimation of the azimuth and elevation DOAs of point signal sources in the three-dimensional space has also been considered in [e.g., 4, 5] by extending (2) as

\[
y(t) = A(\bar{\phi}, \bar{\theta})s(t) + n(t),
\]

In (4), \( \bar{\theta} = [\theta_1, \theta_2, ..., \theta_L]^T \) is the elevation DOA vector and \( A(\bar{\phi}, \bar{\theta}) = [a(\phi_1, \theta_1), a(\phi_2, \theta_2), ..., a(\phi_L, \theta_L)] \), with \( a(\phi_k, \theta_k), k = 1, 2, ..., L \), the two-dimensional steering vectors.

Recently, the azimuth DOA estimation problem is considered under distributed source models [6]–[8]. For example, consider the distributed source model in [8]. The array output vector \( y(t) \) can be expressed as

\[
y(t) = \frac{1}{2\pi} \int_0^{2\pi} a(\phi)s(\phi, t)d\phi + n(t),
\]

where the distributed source density \( s(\phi, t) \) is temporally and spatially uncorrelated with \( n(t) \). Note that the special case \( s(\phi, t) = \sum_{k=1}^{L} s_k(t) \delta(\phi - \phi_k) \) of (5) represents \( L \) point sources of (1).

In this paper, as an extension of the investigation in [8] and as a complement to that in [9], we consider estimation of the azimuth and elevation angles in the three-dimensional space under a distributed signal source model using two linear arrays. Distributed sources can largely be divided [8] into 1) parametric distributed sources defined with two parameters, the center angle and degree of dispersion, and 2) non-parametric distributed sources which cannot generally be defined with a finite number of parameters. Distributed sources can be described by a distributed source density or a directionality which indicates the amount of source power coming from each direction as a continuum. In this paper, we concentrate on the three-dimensional DOA estimation problem on the parametric distributed source density.

Denoting the three-dimensional parametric distributed source density by \( s(\phi, \theta, t) \), we have

\[
s(\phi, \theta, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} g_{mn}(t)e^{-im\phi}e^{-jn\theta},
\]

where \( g_{mn}(t) \) is a random variable with \( E[g_{mn}(t)] = 0 \) and \( E[g_{mn}(t)g_{m'n'}(t)] = \gamma_{mnkl} \). The expansion (6) is possible since \( s(\phi, \theta, t) \) is a periodic function of \( \phi \) and \( \theta \). Then, we have the covariance function \( R_s \) of the signal source as

\[
R_s(\phi, \theta, \phi', \theta') = E[s(\phi, \theta, t)s^*(\phi', \theta', t)] = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \gamma_{mnkl}e^{-j(m\phi-k\phi'+n\theta-l\theta')},
\]

If the signal sources are point sources, the covariance function \( R_s \) has peaks at the DOAs and the magnitudes are theoretically infinite. On the other hand, when the signal source density is distributed, the covariance function takes on large values around the center directions. Under this distributed source density formulation, the output \( y(t) \) of an array can be expressed as

\[
y(t) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-2\pi}^{2\pi} a(\phi, \theta)s(\phi, \theta, t)d\phi d\theta + n(t).\]

The expression (8) is a generalization of (4) and an extension of (5). If the \( L \) signal sources are impulse-like so that

\[
s(\phi, \theta, t) = 4\pi^2 \sum_{k=1}^{L} s_k(t) \delta(\phi - \phi_k) \delta(\theta - \theta_k),
\]

then we obtain the two dimensional point source model of (4) from (8).

2.2 The Parametric Distributed Source Model

As is evident from (8), it is very difficult to proceed further for arbitrary three-dimensional distributed source densities, unless some restrictions are imposed on the characteristics of \( s(\phi, \theta, t) \). In this paper, to obtain tangible results, we will consider a sub-class of the distributed sources.

Let \( g_{mn}(t) = \sum_{k=1}^{L} s_k(t) \rho_k^m e^{im\phi_k} \eta_k^n e^{jn\theta_k} \), where
\( \phi_k \) and \( \theta_k \) are the center angles (which will be called the DOAs in this paper) and \( \rho_k \) and \( \eta_k \) are the dispersion parameters (DPs) with \( 0 < \rho_k, \eta_k < 1 \), \( 0 \leq \phi_k < 2\pi \), and \( 0 \leq \theta_k < \pi \). Note that in \([6]\) and \([8]\), similar restrictions have also been imposed to get physical and analytic results to any degree. As we have noted in Introduction, there exist other models for distributed sources \([10], [11]\) together with some physical insights. For the model of this paper, such physical insights have been given in \([8]\): although the description in \([8]\) is for the two-dimensional case, it is rather directly applicable to the three-dimensional case considered in this paper.

We will call \( s_k(\phi, \theta, t) = s_k(t) \sum_{m=0}^{\infty} \rho_k^m e^{-jm(\phi - \phi_k)} \eta_k^n e^{-n(\theta - \theta_k)} \) the two-dimensional case, it is rather directly applicable sources \([10], [11]\) together with some physical insights.

For the model of this paper, such physical insights have been given in \([8]\): although the description in \([8]\) is for the two-dimensional case, it is rather directly applicable to the three-dimensional case considered in this paper.

Now, the distributed source density composed of \( L \) parametric sources can be expressed as

\[
s(\phi, \theta, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \eta_k^n e^{-n(\theta - \theta_k)} \sum_{k=1}^{L} s_k(t) \sum_{m=0}^{\infty} \rho_k^m e^{-jm(\phi - \phi_k)} = \sum_{k=1}^{L} s_k(t) I_k(\phi, \theta),
\]

(10)

where \( I_k(\phi, \theta) = I(\phi; \theta; \phi_k, \rho_k, \theta_k, \eta_k) = 1/[(1 - \rho_k e^{-j(\phi - \phi_k)})(1 - \eta_k e^{-j(\theta - \theta_k)})] \) will be called the intensity function of the \( k \)th source, with \( I(\phi; \theta; \phi', \rho', \theta', \eta') = 1/[(1 - \rho e^{-j(\phi - \phi')})(1 - \eta e^{-j(\theta - \theta')})] \). The array output of the parametric distributed sources with density (10) can be written as

\[
y(t) = \sum_{k=1}^{L} \frac{s_k(t)}{4\pi^2} \int_{0}^{\pi} \int_{0}^{2\pi} a(\phi, \theta) I_k(\phi, \theta) d\phi d\theta + n(t).
\]

The array output vector in (11) can be obtained by the Cauchy integration once \( a(\phi, \theta) \) is specified, which depends on the array structure. Note that (11) becomes (4) when all \( \rho_k, \eta_k \to 1 \), since

\[
y(t) = \sum_{k=1}^{L} \frac{s_k(t)}{4\pi^2} \int_{0}^{\pi} \int_{0}^{2\pi} a(\phi, \theta) d\phi d\theta + n(t).
\]

In this section, we consider a method to obtain the DOA and DP from the covariance matrices obtained with two perpendicular uniform linear arrays shown in Fig. 1. From (11), we have

\[
\hat{R}_y = \frac{1}{N} YY^H,
\]

(13)

where \( Y = [y(t_1), y(t_2), \ldots, y(t_N)] \) is an \( M \times N \) matrix. We then find the null spectrum by using, for example, the MUSIC-like eigen-decomposition method with the sample covariance function of the output. In other words, the set of the four parameters, \( \phi_k, \rho_k, \theta_k, \eta_k \), for \( k = 1, 2, \ldots, L \), can be estimated as

\[
\hat{[\phi, \rho, \theta, \eta]} = \text{arg} \max_{[\phi, \rho, \theta, \eta]} V(\phi, \rho, \theta, \eta)
\]

(14)

where \( \hat{\phi} = [\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_L], \hat{\rho} = [\hat{\rho}_1, \hat{\rho}_2, \ldots, \hat{\rho}_L], \hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_L], \) and \( \hat{\eta} = [\hat{\eta}_1, \hat{\eta}_2, \ldots, \hat{\eta}_L] \) are the estimate vectors, and \( V \) is a null spectrum which depends on the array used in the estimation.

In short, when the signal sources are point sources, the covariance function of \( s(\phi, \theta, t) \) has peaks at the DOAs and their magnitudes are theoretically infinite. Although the point signal source model sometimes provides a good approximation to real signal sources, the distributed source model can provide a more reasonable approximation when the signal source is not a point source but is distributed over an area in the three-dimensional space. In Sect. 3, we will describe in detail the distributed DOA estimation method with two (that is, azimuth and elevation) perpendicular linear arrays.

In Sect. 4, numerical examples and results will be provided.

### 3. Estimation with Two Linear Arrays

In this section, we consider a method to obtain the DOA and DP from the covariance matrices obtained with two perpendicular uniform linear arrays shown in Fig. 1.
the steering vectors $v$.

\[
\psi = e^{j\theta} e^{-j\phi} + 1, \quad \eta = e^{j\theta} e^{-j\phi} + 1
\]

where $\psi$ and $\eta$ are the phase shift factors for the horizontal and vertical arrays, respectively. For notational convenience, the $i$th element of the column vector $\mathbf{b}_{h,k}$ in the parametric distributed source model is, using the Cauchy integral formula,

\[
\mathbf{b}_{h,k} = \mathbf{b}_h(\phi, \rho, \theta, \eta) = \mathbf{b}_v(\theta, \eta),
\]

and

\[
\mathbf{b}_v(\theta, \eta) = \frac{1}{2\pi} \int_{0}^{2\pi} \frac{a_v(\zeta)}{1 - \eta e^{-j(\xi - \theta)}} d\zeta.
\]

In (15)–(18), the subscripts $h$ and $v$ of $y, b$, and $a$ are used to denote the horizontal and vertical arrays, respectively. Specific expressions of $a_h(\phi, \theta)$ and $a_v(\theta)$ for uniform linear arrays are shown in Appendix A. It is easy to see that $b_{v,k}$ and $b_{h,k}$ are the averaged versions of the steering vectors $a_r$ and $a_h$. Since the covariance function of $s(\phi, \theta, t)$ in the parametric distributed source model is

\[
R_s(\phi, \theta, \phi', \theta') = E[s(\phi, \theta, t)s^*(\phi', \theta', t)]
\]

the covariance matrices of the horizontal and vertical array outputs are

\[
R_{h,y} = B_h \Delta B_h^H + \sigma^2 I
\]

and

\[
R_{v,y} = B_v \Delta B_v^H + \sigma^2 I,
\]

where $[\Delta]_{mn} = [p_{mn}] = E[s_m(t)s_n^*(t)]$ for $m, n = 1, 2, \ldots, L$. $B_h = [b_{h,1}, b_{h,2}, \ldots, b_{h,L}]$, and $B_v = [b_{v,1}, b_{v,2}, \ldots, b_{v,L}]$.
\[ S_v = [e_{v,1}, e_{v,2}, \cdots, e_{v,L}] \] and \[ G_v = [e_{v,L+1}, e_{v,L+2}, \cdots, e_{v,M}] \], where \( e_{v,k} \) is the eigenvector corresponding to the \( k \)th largest eigenvalue \( \lambda_{v,k} \) of \( R_{v,y} \) with \( \lambda_{v,1} > \lambda_{v,2} > \cdots > \lambda_{v,L} = \lambda_{v,L+1} = \cdots = \lambda_{v,M} \). Similarly, let \( S_h = [e_{h,1}, e_{h,2}, \cdots, e_{h,L}] \) and \( G_h = [e_{h,L+1}, e_{h,L+2}, \cdots, e_{h,M}] \), where \( e_{h,k} \) is the eigenvector corresponding to the \( k \)th largest eigenvalue \( \lambda_{h,k} \) of \( R_{h,y} \) with \( \lambda_{h,1} > \lambda_{h,2} > \cdots > \lambda_{h,L} > \lambda_{h,L+1} = \lambda_{h,L+2} = \cdots = \lambda_{h,M} \). Practically, the estimation of the parameters \( \theta, \eta \), \( \phi, \rho \), \( \lambda \) can be found using the following orthogonality properties:

\[
b_v^H(\theta, \eta) G_v = 0
\]

when \( (\theta, \eta) \in \{(\theta_1, \eta_1), (\theta_2, \eta_2), \cdots, (\theta_L, \eta_L)\} \)

and

\[
b_v^H(\phi, \rho, \theta, \eta) G_v = 0
\]

when \( (\phi, \rho, \theta, \eta) \in \{(\phi_1, \rho_1, \theta_1, \eta_1), \cdots, (\phi_2, \rho_2, \theta_2, \eta_2), \cdots, (\phi_L, \rho_L, \theta_L, \eta_L)\} \).

Practically, the estimation of the parameters \( (\phi_k, \rho_k, \theta_k, \eta_k), k = 1, 2, \cdots, L \), can be implemented in several ways: we describe here one of the possible procedures.

**L1** Estimation of \( \theta_k \) and \( \eta_k \), \( k = 1, 2, \cdots, L \):

Since these parameters are independent of the horizontal parameters \( \phi_k \) and \( \rho_k \), we can use the conventional MUSIC-like null spectrum

\[
f_v(\theta, \eta) = \frac{||b_v(\theta, \eta)||^2}{b_v^H(\theta, \eta) \hat{G}_v G_v^H b_v(\theta, \eta)},
\]

where \( \hat{G}_v \) is the noise subspace of the sample covariance function of the vertical output array. With (26) we have

\[
(\hat{\theta}_k, \hat{\eta}_k) = \arg \max_{\theta, \eta} f_v(\theta, \eta),
\]

for \( k = 1, 2, \cdots, L \).

**L2** Estimation of \( \phi_k \) and \( \rho_k \), \( k = 1, 2, \cdots, L \):

With the estimates \( \hat{\theta}_k \) and \( \hat{\eta}_k \) obtained in (L1), we can estimate the parameters \( \phi_k \) and \( \rho_k \), \( k = 1, 2, \cdots, L \) by using the two dimensional maximization process as

\[
(\hat{\phi}_k, \hat{\rho}_k) = \arg \max_{\phi, \rho} f_{h,k}(\phi, \rho, \hat{\theta}_k, \hat{\eta}_k),
\]

where

\[
f_{h,k}(\phi, \rho, \hat{\theta}_k, \hat{\eta}_k) = \frac{||b_h(\phi, \rho, \hat{\theta}_k, \hat{\eta}_k)||^2}{b_h^H(\phi, \rho, \hat{\theta}_k, \hat{\eta}_k) \hat{G}_h G_h^H b_h(\phi, \rho, \hat{\theta}_k, \hat{\eta}_k)}.\]

4. Numerical Examples and Results

In this section, to see the developments in the previous sections more explicitly, we will consider some parameter estimation examples and results when \( L = 2 \) and \( M = 10 \): the two linear arrays shown in Fig. 1 are uniform each with \( M/2 = 5 \) elements.

Consider the two equal-power uncorrelated distributed sources with parameter value sets

\[
(\phi_1, \rho_1, \theta_1, \eta_1) = (30^\circ, 0.9, 65^\circ, 0.8)
\]

and

\[
(\phi_2, \rho_2, \theta_2, \eta_2) = (40^\circ, 0.7, 50^\circ, 0.6),
\]

for which the intensity functions \( I_1(\phi, \theta) \) and \( I_2(\phi, \theta) \) are shown in Figs.2 and 3, respectively. Here, the SNR is defined as 10 log \[ \frac{|E|}{\sigma^2} \] (dB).

**Example 1:**

To see the necessity of the parametric distributed source model, we obtained the conventional MUSIC null spectrum

\[
f_1(\theta) = \frac{||a_v(\theta)||^2}{a_v^H(\theta) \hat{G}_v G_v^H a_v(\theta)}
\]

of the vertical array under the point source assumption at SNR = 15 dB, as shown in Fig. 4. In this figure, only one maximum point around \( 90^\circ - \hat{\theta} = 30^\circ \) can be found, and thus the other source cannot be located. This example shows that the MUSIC-based estimator may fail when depending upon use of a point source assumption in the parametric distributed source model.

**Example 2:**

First assume that \( \hat{G}_v = G_v \) and \( \hat{G}_h = G_h \). Then consider the null spectra (26) and (29). At SNR = 15 dB, the maximization process (27) with the null spectrum (26) shown in Fig. 5 and the contour shown in Fig. 6.
Fig. 3  The intensity function of the distributed source 
\((40^\circ, 0.7, 50^\circ, 0.6)\).

Fig. 4  Conventional MUSIC null spectrum with two linear 
arrays.

Fig. 5  The null spectrum \(f_v(\theta, \eta)\) with \(\hat{G}_v = G_v\).

results in \((90^\circ - \hat{\theta}_1, \hat{\eta}_1) = (25^\circ, 0.8)\) and \((90^\circ - \hat{\theta}_2, \hat{\eta}_2) = 
(40^\circ, 0.6)\).

Using these estimates in the maximization process 
(28), we can next obtain \((\hat{\phi}_1, \hat{\rho}_1)\) from Figs. 7 and 8, and 
\((\hat{\phi}_2, \hat{\rho}_2)\) from Figs. 9 and 10. That is, once \((\hat{\theta}_1, \hat{\eta}_1)\) is ob-
tained, we can use the values \((90^\circ - \hat{\theta}_1, \hat{\eta}_1) = (25^\circ, 0.8)\) to 
 obtain \((\hat{\phi}_1, \hat{\rho}_1)\) of the first source from (28) and (29),
as shown in Figs. 7 and 8. Next, to obtain the horizontal 
(azimuth) parameters \((\hat{\phi}_2, \hat{\rho}_2)\) of the second source, 
we use the values \((90^\circ - \hat{\theta}_2, \hat{\eta}_2) = (40^\circ, 0.6)\) in (28) and 
(29), and obtain the null spectrum \(f_{h,2}(\phi, \rho, \hat{\theta}_2, \hat{\eta}_2)\) as 
shown in Fig. 9: the contour used for the maximization 
process (28) is shown in Fig. 10.
Example 3:
We set the number of snapshots $N = 100$. The noise subspaces $\hat{G}_v$ and $\hat{G}_h$ are obtained through the eigen-decomposition of the sample covariance functions $R_{v,y} = \frac{1}{N} Y_v Y_v^H$ and $\hat{R}_{h,y} = \frac{1}{N} Y_h Y_h^H$ obtained based on random number generation of the two vectors $s(t) = [s_1(t), s_2(t), \cdots, s_L(t)]^T$ and $n(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T$. Here $Y_v = [y_v(t_1), y_v(t_2), \cdots, y_v(t_N)]$ and $Y_h = [y_h(t_1), y_h(t_2), \cdots, y_h(t_N)]$ are $M \times N$ matrices of the vertical and horizontal array outputs, respectively. We have estimated the two sets of the four parameters by implementing Procedures (L1) and (L2) when the SNR = 20dB, 15dB, and 10dB. When the SNR = 20dB, Figs. 11 and 12 show the implementations of (26) and (27) for the estimation of the two vertical parameters. Figures 13 and 14 show the implementations of (29) and (28) for estimating the two horizontal parameters of the first source; the implementations of (29) and (28) for estimating the two horizontal parameters of the second source are shown in Figs. 15 and 16. We finally get $(\phi_1, \rho_1, 90^\circ - \hat{\theta}_1, \hat{\eta}_1) = (30.5^\circ, 0.9, 25^\circ, 0.79)$ and $(\phi_2, \rho_2, 90^\circ - \hat{\theta}_2, \hat{\eta}_2) = (41^\circ, 0.75, 40^\circ, 0.6)$. Although we do not show explicitly in this paper, similar results are obtained for the cases SNR = 15dB and 10dB.

5. Conclusion
The problem of direction of arrival estimation has been studied for some decades as an important topic in the area of array signal processing. In this paper, we have addressed the estimation of the two-dimensional (azimuth and elevation) direction of arrival based on a parametric distributed source model.
In several physical environments, signal source localization should be performed based on a distributed source model in the three-dimensional space. When the signal sources are not point sources, but dispersed over an area, we cannot directly use the well-known direction of arrival estimation methods, because these methods are established based on the point source assumption.

In this paper, we have proposed a three-dimensional distributed signal source model which are represented with the center angles and degrees of dispersion. We have investigated some eigenstructure-based algorithms to estimate the locations of the two-dimensional distributed sources. We have shown that the performance of the method is acceptable when the signal sources are distributed while the conventional method based on the point source model fails to correctly locate the sources.

It is noteworthy that the results presented in this paper complement those in [9], where circular arrays are used for DOA estimation.

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References

Appendix

From Fig. 1, we can obtain the steering vector $a_v$ of the vertical array as

$$a_v(\theta) = \begin{bmatrix} e^{j \cos \theta}, e^{j 2 \cos \theta}, \ldots, e^{j M \cos \theta} \end{bmatrix}^T,$$

(A.1)

where $\theta$ is the elevation angle of a signal source. Similarly, the steering vector for the horizontal array can be obtained as

$$a_h(\phi, \theta) = \begin{bmatrix} e^{j \cos \phi \cos \theta}, e^{j 2 \cos \phi \cos \theta}, \ldots, e^{j M \cos \phi \cos \theta} \end{bmatrix}^T,$$

(A.2)

where $\phi$ and $\theta$ are the azimuth and elevation angles of a signal source, respectively.

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