Symbol Error Rate of the DM-MPSK Under the Influence of Jamming Signals

Youngpo Lee, Student Member, IEEE, Taecung Yoon, Student Member, IEEE, Iickho Song, Fellow, IEEE, Yun Hee Kim, Senior Member, IEEE, Tae Hee Han, Member, IEEE, and Seokho Yoon, Senior Member, IEEE

Abstract—Focusing on the performance of the chirp spread spectrum (CSS) communication system under the influence of jamming, we analytically derive the symbol error rate (SER) for the direct modulation CSS system with M-ary phase shift keying in the presence of broadband and tone jamming signals. From numerical results, it is confirmed that the analytic result agrees closely with the empirical SER.

Index Terms—CSS, DM, MPSK, SER, jamming.

I. INTRODUCTION

In chirp spread spectrum (CSS) techniques, the data signal is spread over a wider frequency band via chirp signals for transmission. Because of its capability to resist jamming, CSS techniques have recently attracted much attention in the field of wireless communications [1], and have been adopted as a physical layer implementation of IEEE 802.15.4a, the standard for low-rate wireless personal area networks [2].

Depending on the role of the chirp signal in the modulation process, the CSS technique is classified into two categories, the binary orthogonal keying (BOK) and direct modulation (DM) [3]. The BOK scheme uses chirp signals for representing data: for example, bits ‘1’ and ’0’ can be represented by chirp signals with positive and negative instantaneous frequency change rates, respectively. On the other hand, the chirp signal in the DM scheme is used just as a spreading code and data modulation and demodulation is performed separately from the chirp processing, allowing the DM scheme a flexibility to incorporate various modulation techniques.

In this paper, we analyze the symbol error rate (SER) performance of the DM scheme with M-ary phase shift keying (DM-MPSK) under the influence of jamming. Specifically, we focus on the SER performance in the presence of broadband and tone jamming signals, of which the former jams the entire bandwidth of the transmitted signal while the latter affects a specific frequency (mainly, the carrier frequency) of the transmitted signal.

Fig. 1. The complex baseband model of the DM-MPSK system.

II. SYSTEM MODEL

The complex baseband equivalent $c(t)$ of a chirp waveform can be expressed as

$$c(t) = \sqrt{\frac{T}{T_c}} \exp(j\pi\mu t^2), \quad |t| < \frac{T_c}{2},$$

where $T_c$ denotes the duration of the chirp signal. In (1), the non-zero parameter $\mu$, called the chirp rate, denotes the instantaneous frequency change rate of the chirp signal and defines the CSS bandwidth $B$ by

$$B = |\mu|T_c. \quad (2)$$

When $\mu > 0$ ($\mu < 0$), the chirp signal is called the up-chirp (down-chirp) signal and the instantaneous frequency increases (decreases). In this paper, we assume $\mu > 0$ for simplicity.

Fig. 1 shows the complex baseband model of the DM-MPSK system. Input data is first mapped into MPSK constellations, and subsequently, multiplied by an up-chirp signal $c(t - iT_c)$. Then, the $i$-th DM-MPSK symbol $s_i(t)$ is

$$s_i(t) = \sqrt{E_s} e^{j\phi_i} c(t - iT_c), \quad |t - iT_c| < \frac{T_c}{2} \quad (3)$$

for $i = 0, \pm 1, \pm 2, \ldots$, where $E_s$ is the symbol energy and $\phi_i$ denotes the $i$-th data taking a value in $\{0, \frac{2\pi}{M}, \ldots, \frac{2(M-1)\pi}{M}\}$ with probability $1/M$. The DM-MPSK symbol is contaminated by a jamming signal $z(t)$ during transmission. At the receiver, the contaminated DM-MPSK symbol is correlated with a down-chirp signal $c^*(t - iT_c)$; thus, the $i$-th correlator output $g_i$ can be expressed as

$$g_i = \int_{iT_c - \frac{T_c}{2}}^{iT_c + \frac{T_c}{2}} \{s_i(t) + z(t)\} c^*(t - iT_c) dt$$

$$= \sqrt{E_s} e^{j\phi_i} \int_{iT_c - \frac{T_c}{2}}^{iT_c + \frac{T_c}{2}} z(t) c^*(t - iT_c) dt. \quad (4)$$
Finally, the output data is determined by demapping and detecting the correlator output $g_0$. Based on the fact that the jamming power is generally much larger than the power of thermal noise [4], and assuming that the optimum decision rule for MPSK against the noise is employed, we disregard the effect of noise as in [5].

### III. SYMBOL ERROR RATE OF THE DM-MPSK

#### A. In Broadband Jamming Environment

In the broadband jamming model, the jamming signal $z(t)$ can be modeled as a zero-mean Gaussian process with a flat power spectral density over the frequency range of interest [4]. Thus, the SER of the DM-MPSK system under the influence of a broadband jamming is identical to that in additive white Gaussian noise (AWGN) channels. In addition, when the chirp signal is used only as a spreading code and the bandwidth of the DM-MPSK system is infinite, the effect of the CSS on the error performance in AWGN channels may be ignored [6]. Thus, from the SER expression of the MPSK over AWGN channels [4], we can easily obtain the SER

$$P_{BJ} = I_M Q \left( \sqrt{\frac{2E_s}{J_0}} \sin \frac{\pi}{M} \right)$$

of the DM-MPSK under the influence of broadband jamming, where $I_M = 1$ and 2 for $M = 2$ and $M > 2$, respectively, $J_0$ denotes the power spectral density of the jamming signal, and $Q(x) = \frac{1}{\sqrt{\pi}} \int_x^\infty \exp \left( -\frac{t^2}{2} \right) dt$.

In practical DM-MPSK systems with finite bandwidth, the parameter $J_0$ in (5) should be replaced by $\alpha J_0$ with $\alpha$ a value slightly smaller than 1. For example, we can obtain $\alpha = 0.977$ after some manipulations when the system bandwidth of DM-BPSK is equal to the CSS bandwidth $B = |\mu| T_c$ defined in (2). Interestingly, $\alpha = 0.903$ is reported [6] for direct sequence spread spectrum (DSSS) employing a pseudo-noise (PN) spreading code and BPSK when the system bandwidth is equal to the null-to-null main-lobe bandwidth.

#### B. In Tone Jamming Environment

In the tone jamming model, a jamming signal can be represented as $\text{Re}\{\sqrt{J} e^{j\psi} e^{j2\pi f,t}\}$, where $\text{Re}\{\cdot\}$ denotes the real part, $J = BJ_0$ is the jamming power, $\theta$ is a random variable distributed uniformly over $[0, 2\pi]$, and $f_c$ is the carrier frequency of the transmitted signal [7]. In this paper, we focus on the worst-case where the tone jamming frequency is the same as the carrier frequency. Denoting the complex baseband equivalent of the tone jamming signal by

$$z(t) = \sqrt{J} \exp(j\theta),$$

the correlator output for the tone-jammed DM-MPSK symbol can be expressed as, letting $i = 0$ without loss of generality,

$$g_0 = \sqrt{E_s} e^{j\psi_0} + \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} z(t)e^{*}(t)dt \quad \text{from (4). Observing that the integral in (7) is in the form of Fresnel integral }$$

$$g_0 = \sqrt{E_g} e^{j\psi_0} + \sqrt{2J_0} R_e e^{j\psi}$$

after some manipulations. In (8),

$$R = \left\{ X^2 \left( \sqrt{G_p/2} \right) + Y^2 \left( \sqrt{G_p/2} \right) \right\}^{\frac{1}{2}}$$

is the ‘amplitude’ and

$$\psi = \theta + \angle \left\{ X \left( \sqrt{G_p/2} \right) - jY \left( \sqrt{G_p/2} \right) \right\}$$

is the ‘phase’ distributed uniformly over $[0, 2\pi]$, with

$$X(v) = \int_0^v \cos \left( \frac{\pi v^2}{2} \right) dv,$$

$$Y(v) = \int_0^v \sin \left( \frac{\pi v^2}{2} \right) dv,$$

and $G_p = BT_c = \mu T_c^2$ the processing gain of the DM scheme.

Depending on the value of $E_s/J_0$, (8) furnishes three distinct signal space representations of $g_0$ as shown in Fig. 2, where the trace of $g_0$ due to the jamming vector (with different $\psi$) is delineated with the dotted line assuming $\psi_0 = 0$. Using
Fig. 2, we can analytically derive the SER of the DM-MPSK under the influence of tone jamming signals. Specifically, since a symbol error occurs if $g_0$ falls on the outside of the decision boundary, the SER caused by tone jamming can be obtained as

$$\frac{\Theta_1 + \Theta_2}{\pi} = \frac{M-2}{2M} + \frac{1}{\pi} \cos^{-1} \left( \sqrt{\frac{E_{b,1}}{2J_0} \frac{1}{M} \sin \frac{\pi}{M} \left( \sqrt{\frac{E_{b,2}}{2J_0}} \right)} \right)$$

when $E_{b,2} = 0$, as shown in Fig. 2(a). When $R^2 < \frac{E_{b,2}}{2J_0} \leq \frac{R^2}{\sin^2(\pi/M)}$, the SER can easily be obtained as

$$2 \pi \cos^{-1} \left( \sqrt{\frac{E_{b,1}}{2J_0} \frac{1}{M} \sin \frac{\pi}{M} \left( \frac{R^2}{\sin^2(\pi/M)} \right)} \right),$$

from Fig. 2(b). Finally, the SER is zero when $\frac{E_{b,2}}{2J_0} > \frac{R^2}{\sin^2(\pi/M)}$, since the entire trace of $g_0$ lies within the decision boundary as shown in Fig. 2(c).

In summary, we have obtained the analytic expression

$$P_{T,J} = \begin{cases} \frac{M-2}{2M} + \frac{1}{\pi} \cos^{-1} \left( \sqrt{\frac{E_{b,1}}{2J_0} \frac{1}{M} \sin \frac{\pi}{M} \left( \sqrt{\frac{E_{b,2}}{2J_0}} \right)} \right), \
\frac{2\pi}{\pi} \cos^{-1} \left( \sqrt{\frac{E_{b,1}}{2J_0} \frac{1}{M} \sin \frac{\pi}{M} \left( \sqrt{\frac{E_{b,2}}{2J_0}} \right)} \right), \
0, \end{cases}$$

(13)

of the SER of the DM-MPSK under the influence of tone jamming signals. Note that, when $M = 2$, the second line of (13) disappears since $R^2 < \frac{E_{b,2}}{2J_0} \leq \frac{R^2}{\sin^2(\pi/2)}$ denotes an empty set for $\frac{E_{b,2}}{2J_0}$. In passing, we would like to mention that, although the performance in tone jamming was analyzed in [6] for DS/SS employing PN spreading codes, the approach in [6] is not applicable in the performance analysis of CSS systems in tone jamming. This is because the Gaussian approximation of the correlator output exploiting the statistical property of PN codes, on which the approach in [6] is based, is not applicable when a chirp signal is employed as the spreading code.

### C. Numerical Results

Assuming $T_c = 0.5 \mu s$, $\mu = 400 \text{MHz/\mu s}$, $B = 200 \text{MHz}$, and $G_p = 100$, let us now compare the theoretical SER (13) of the DM-MPSK derived in this paper with the empirical SER based on Monte Carlo simulation. In the comparisons, DM-BPSK ($M = 2$), DM-QPSK ($M = 4$), and DM-8PSK ($M = 8$) are considered.

Fig. 3 shows the SER of the DM-MPSK in tone jamming, where the theoretical SER curves are evaluated using (13): here, $E_b = \frac{E_b}{2J_0}$ is the bit energy. The figure clearly exhibits a close agreement between the theoretical and simulated SER results. It is also observed that no symbol error due to jamming occurs when $\frac{E_{b,2}}{2J_0} > \frac{R^2}{\sin^2(\pi/M)}$, as anticipated. Specifically, for $G_p = 100$, the threshold $\frac{E_{b,2}}{2J_0}$ for zero SER is $10 \log \left( \frac{E_{b,1}}{2J_0} \right) \approx 392 \text{dB}$, $392 \text{dB}$, and $395 \text{dB}$ when $M = 2, 4, \text{ and } 8$, respectively.

Since the SER of the DM-MPSK in broadband jamming is the same as that of the MPSK over the AWGN channels shown in [4], we have not included any figure; yet, the theoretical SER from (5) is observed to agree with the empirical SER.

### IV. Conclusion

We have analyzed the SER performance of the DM-MPSK system under the influence of broadband and tone jamming signals. Modeling the broadband jamming as a Gaussian process, we have first obtained the SER expression for the DM-MPSK under the influence of broadband jamming. Then, the SER of the DM-MPSK under the influence of tone jamming has been derived based on the geometric analysis of the signal space representation of the correlator output: in general, there exist three distinct expressions depending on the value of the signal-to-jamming ratio. Finally, we have compared the analytic SER derived in this paper with the empirical SER. From the numerical results, it has been confirmed that the analytic and empirical results agree closely with each other.

### REFERENCES


